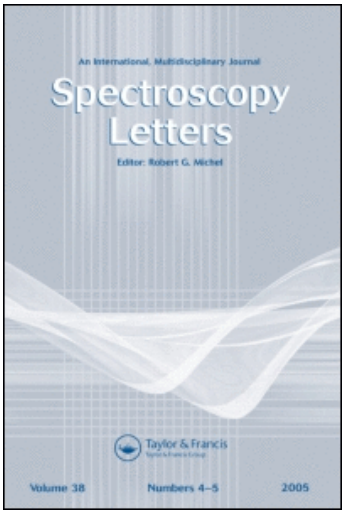

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The Interacting Boson Approximation Model of the Deformed Nuclei $^{158-162}$Dy, $^{182-186}$W, and ^{158}Gd

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THE INTERACTING BOSON APPROXIMATION MODEL OF THE DEFORMED NUCLEI $^{158-162}Dy$, $^{182-186}W$, AND ^{158}Gd

Key words: The interacting boson approximation, the SU(3) limit, PHINT parameters, deformed nuclei.

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ABSTRACT

The low-lying energy levels of the deformed $^{158-162}Dy$, $^{182-186}W$ and ^{158}Gd nuclei were obtained by using the IBA-1 model. In these nuclei, it is necessary to have the symmetry breaking terms for the energy levels calculated according to the SU(3) limit to be in better agreement with the experimental data. Therefore, a small symmetry breaking term was added to SU(3) Hamiltonian in the direction of O(6) limit. It was seen that these newly obtained energy levels were in good agreement with the experimental data. For these nuclei , B(E2) values were also calculated and compared with the experimental data and the previous studies. It was concluded that the agreement was remarkably good.

INTRODUCTION

The interacting boson approximation (IBA) is a model which was proposed in order to calculate the energy levels of medium and the heavy mass even-even nuclei [1,5]. Most of the properties of even-even nuclei have been investigated using different approaches within the framework of the IBA model [6,7,8]. In the IBA model, the spectra of low-energy collective properties of even-even nuclei were described in terms of a system of interacting L=0 and L=2 bosons (s and d bosons). Furthermore, the model assumed that the structure of low-lying levels was obtained by exciting the valance nucleons outside major closed shell. The number of proton bosons, N_π, and the number of neutron bosons, N_ν were counted according to the nearest closed shell, and resulting total boson number, $N=N_\pi+N_\nu$, was a strictly conserved

quantity. The structure of Hamiltonian and states were defined according to U(6) symmetry group. There were three different subgroups to the dynamical symmetries of the spherical vibrator U(5), deformed rotor O(6). The distinctive structures of the three dynamical symmetries in the IBA provided three clear-cut limits of the general Hamiltonian. Although evidence exited which suggested that features of the pure symmetries were observed emprically in selected nuclei, a realistic calculation required a departure from the strict limits or indeed a transition between them.

In this study, the SU(3) symmetries of the deformed $^{158\text{-}162}$Dy , $^{182\text{-}186}$W and ^{158}Gd nuclei have to be broken. Since the energy levels, calculated according to the SU(3) symmetry limit , were not in agreement with the experiments very well, the perturbation was applied to SU(3) in the direction of O(6). The IBA-1 Hamiltonian was diagonalized by using PHINT computer code in order to obtain low-lying energy levels. The values of the Hamiltonian parameters were calculated with the help of experimental data while the boson number was determined in the following way. In Dy isotopes, 66 protons and the number of neutrons ranged from 92(^{158}Dy) to 96(^{162}Dy). The number of valance protons was calculated according to the closed-shell, Z=50. Accordingly N_π was equal to 8 and N_ν was between 5 and 7. Thus the total number of bosons, $N=N_\pi+N_\nu$ took the values of 13 and 15. Similarly, the total number of bosons was between 11 and 13 in $^{182\text{-}186}$W isotopes and was 13 in ^{158}Gd . For all these values, low-lying energies and B(E2) values were calculated and compared to the experimental data.

THE MODEL

In the simplest form of interacting boson approximation-1 (IBA-1) there is no distinction between a proton and neutron. The Hamiltonian (H) can be written in terms of boson creation (d^+) and annihilation (d) operators, such that

$$H = \varepsilon' \sum_m{}' d_m^+ d_m + a_0 P^+ P + a_1 L^2 + a_2 Q^2 + a_3 T_3^2 + a_4 T_4^2 \qquad (1)$$

where $\varepsilon' = \varepsilon_d - \varepsilon_s$ is the boson energy [2]. The terms with the coefficients a_0, a_1, a_2 and a_4 designate the strengths of the pairing, angular momentum, quadrupole, octupole and hexadecupole interaction between the bosons, respectively. In calculations, generally there is an L^2 term in every Hamiltonian and it is always diagonal. Its contribution to the level of spin L is L(L+1). It does not effect the energy differences and wave functions of states which have the same spins. The term T_4^2 can be ignored since

it is not necessary for actual applications. T_3^2 stems from O(5) so that it takes place in the chains of both U(5) and O(6). The ε' term places an important role in U(5) structure while the P^+P term and Q^2 term have important roles in O(6) and SU(3) structures, respectively. According to the SU(3) symmetry limit, the Hamiltonian is given by

$$H = a_1 L^2 + a_2 Q^2 \qquad (2)$$

where L^2 and Q^2 represent the boson quadrupole-quadrupole and angular momentum interactions respectively.

In the IBA-1 the E2 operator T(E2) has the form

$$T(E2) = \alpha_2 [d^+ x\ s + s^+ x\ d]^{(2)} + \beta_2 [d^+ x\ d]^{(2)} \qquad (3)$$

with (s^+, d^+) and (s, d) being the creation and annihilation operator for s and d bosons respectively, while α_2 and β_2 are parameters [2].

In this study, PHINT and FBEM computer codes were used for calculations of IBA-1 [9]. The relations, which relate the parameters in Eq. (1) to the variables in PHINT, are given in Table-1. According to this table Hamiltonian can be written as

$$H = \frac{1}{2} ELL(\mathbf{L}.\mathbf{L}) + \frac{1}{2} QQ(\mathbf{Q}.\mathbf{Q}). \qquad (4)$$

In the above equation, the terms with the coefficient of ELL and QQ represent the angular momentum and quadrupole-quadrupole interactions, respectively. This Hamiltonian gives the exact limit of SU(3).The eigenstates are given by the quantum numbers N, L, M and K, N denotes the total number of bosons. L is the angular momentum while M represents the projection of the angular momentum (L) along the symmetry axis of the nucleus. The parameters in Eq.(4) are the names of the variables in the PHINT computer code. The parameters in the Hamiltonian were gradually varied to produce the best fit to the experimentally determined energy levels. In Figure-1, the energy levels calculated for D_y isotopes according to SU(3) Hamiltonian are shown together with the experimental energy levels. As it can be seen from Figure-1, the energy level of ground-state and γ-band are in good agreement with the experiments. However, a good compatibility cannot be observed in the β-band. In this case, it is obvious

Table-1: Relations between some IBA parameters [10].

Parameters of Eq.(1)	PHINT parameters	Equivalent parameters sometimes used in the Literature
ε'	EPS	ε'
a_0	2 PAIR	2K"
a_1	$\frac{1}{2}$ ELL	2K'
a_2	$\frac{1}{2}$ QQ	-2K
a_3	5 OCD	
a_4	5 HEX	

that SU(3) symmetry will be broken because most of the deformed nuclei can not be defined by pure SU(3) limit. If it is started with the SU(3) and the P^+P pairing term is added to it, it will be passed into the direction of O(6) symmetry. If the Hamiltonian is defined by

$$H = a_2 Q^2 + a_0 P^+ P + a_1 L^2 \tag{5}$$

the effect of this term can easily be observed. Since the term L^2 is a diagonal, it is ignored. Therefore Hamiltonian can be expressed as

$$H = a_2 \left[Q^2 + \frac{a_0}{a_2} P^+ P \right] \tag{6}$$

where the parameter a_2 is only a scale factor. The structure is fully specified by the ratio of a_0/a_2 when the coefficents of spin are taken into account perturbation is proportional with $a_0/4\,a_2$. In largely deformed nuclei, $a_0/4\,a_2$ is aproximately -1, i.e $a_0/4\,a_2 \approx -1$. In this case the Hamiltonian in terms of the parameters in the PHINT computer code can be written as

$$H = \frac{1}{2} \mathrm{ELL}(\mathbf{L}.\mathbf{L}) + \frac{1}{2} \mathrm{QQ}(\mathbf{Q}.\mathbf{Q}) + \mathrm{PAIR}(\mathbf{P}.\mathbf{P}) \tag{7}$$

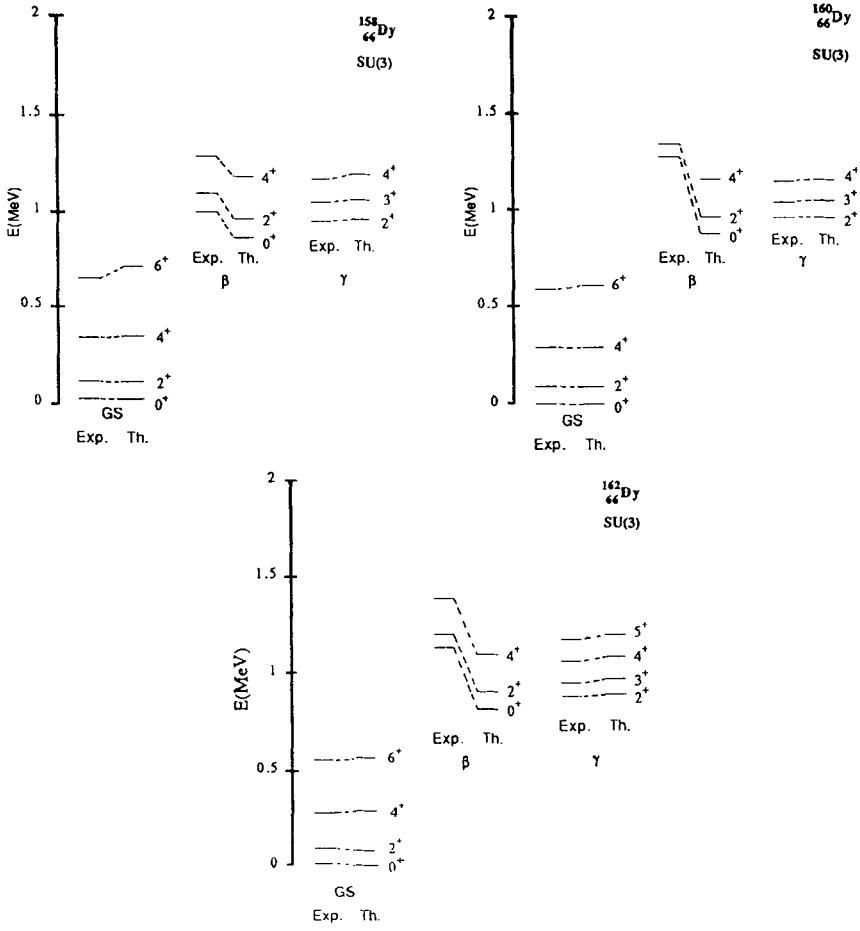

Figure-1: Comparison of the experimental and calculated energy levels of the ground state, β-band and γ-band for the $^{158\text{-}162}$Dy isotopes. Experimental data are taken from [11].

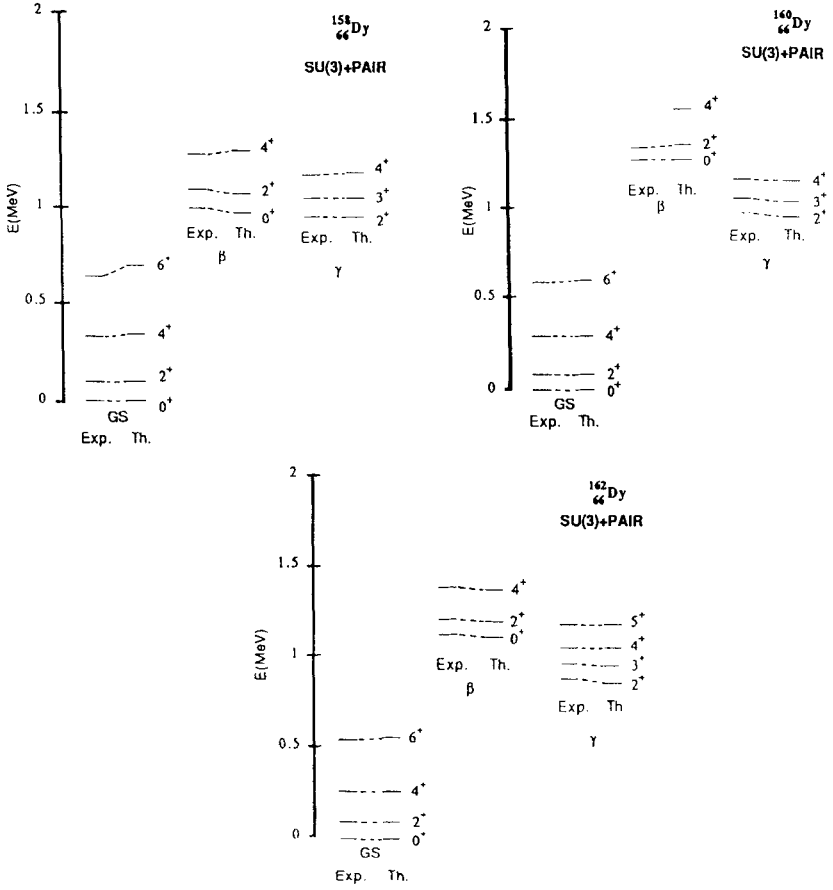

Figure-2: Comparison of the experimental and calculated energy levels of the ground state, β-band, γ-band obtained by adding PAIR pairing term for $^{158\text{-}162}$Dy isotopes. Experimental data are taken from [11].

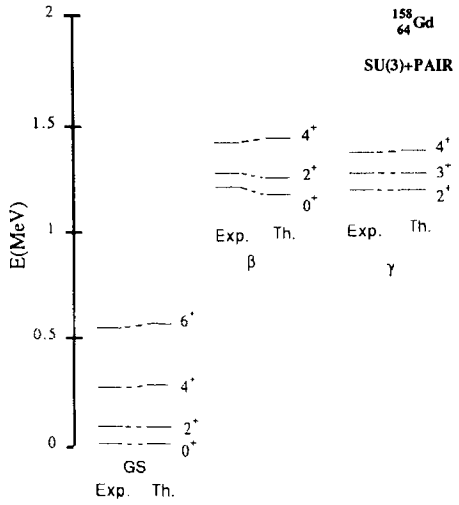

Figure-3: Calculated and experimental energy levels of ground state, β-band, γ-band for the ^{158}Gd nucleus. Experimental data are taken from [11].

It can be concluded from the above equation that perturbation is applied from SU(3) to the direction of O(6). As a result of the above operation $^{158\text{-}162}Dy$ isotopes are obtained as shown in Figure-2. As mentioned previously, the parameters in Eq. (7) are the names of variables used in PHINT. The energy spectrum for ^{158}Gd and $^{182\text{-}186}W$ nuclei obtained by using Hamiltonian given in Eq. (7) are shown in Figures 3 and 4. The values of the parameters of the FBEM and the PHINT computer codes used to calculate the levels of Dy, Gd, and W nuclei are also displayed in Table-2. It is assumed that E2SD = α_2 and E2DD = β_2 . These two parameters were obtained by fitting to the B(E2) values for transitions from the first excited state of the ground-state band and from the lowest state of the γ band to the ground state. In Table-3, the B(E2) values calculated for all these nuclei by using FBEM computer code are compared to the experimental data.

CONCLUSIONS

The deformed nuclei SU(3) whose mass numbers range from 155<A<190 to A≥ 222 generally take place in the region of rotational symmetry. The low-lying energy levels of $^{158\text{-}162}Dy$, $^{182\text{-}186}W$, and

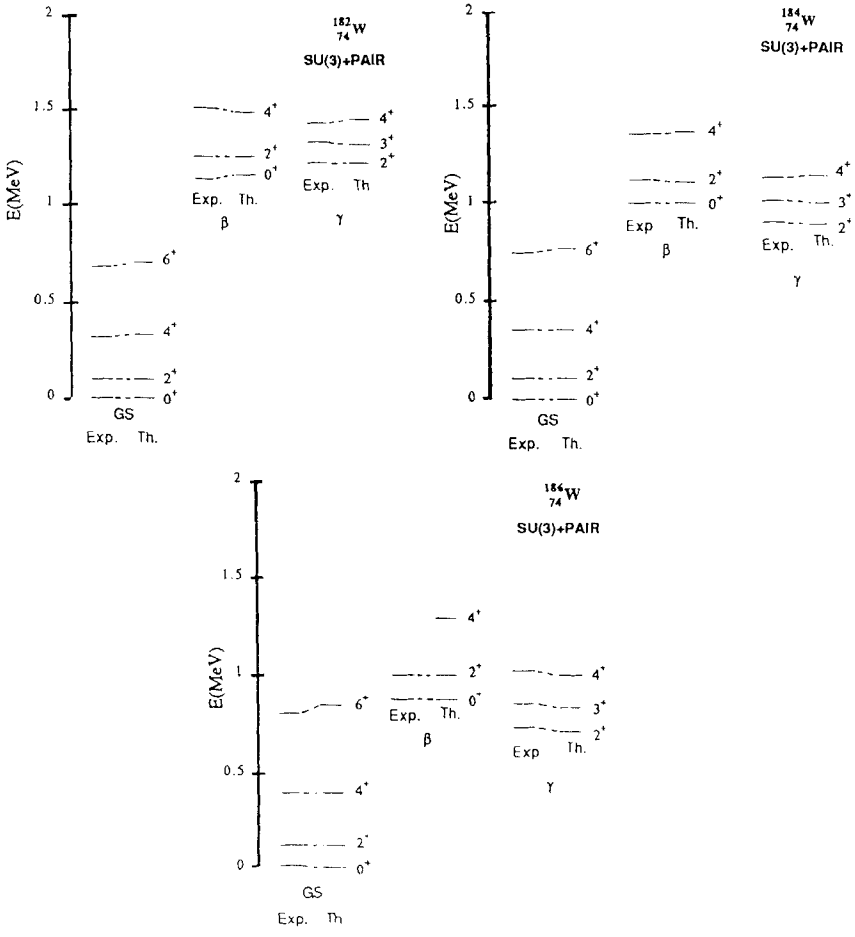

Figure-4: Calculated and experimental energy levels of the ground-state, β-band, γ-band for the 182- 186W nuclei. Experimental data are taken from [11].

Table-2: Interaction parameters of the Hamiltonian for the $^{158\text{-}162}$Dy, $^{182\text{-}186}$W and ^{158}Gd nuclei.

Nucleus	QQ (MeV)	ELL (MeV)	PAIR (MeV)	E2SD (eb)	E2DD (eb)
^{158}Dy	-0.0227	0.0247	0	—	—
	-0.02266	0.02466	0.0055	0.16	0.008
^{160}Dy	-0.021777	0.0208	0	—	—
	-0.0218	0.0208	0.0178	0.1344	-0.1466
^{162}Dy	-0.0186	0.02	0	—	—
	-0.0186	0.02	0.012	0.1289	-0.01233
^{182}W	-0.03	0.022	0.0016	0.0471	-0.671
^{184}W	-0.023	0.02845	0.0105	0.1355	-0.1322
^{186}W	-0.0195	0.0337	0.016	0.1355	-0.1299
^{158}Gd	-0.0296	0.0155	0.0025	0.1433	-0.1433

^{158}Gd nuclei were calculated as well as their B(E2) values in between. SU(3) Hamiltonian for $^{158\text{-}162}$Dy isotopes were used. The interaction parameters in Hamiltonian were determined and the low-lying energy level of Dy isotopes were calculated by using the PHINT computer code. When the calculated energy levels of these nuclei were compared to the experimental energy levels, the ground-state and γ-band were in quite good agreement whereas β-band did not have the desired agreement. In this study, this disagreement was tried to be removed.

In general, most of the deformed nuclei can not be defined within the given SU(3) symmetry limits. The symmetry breaking of SU(3) has to be taken into consideration. A small symmetry breaking

Table-3: Calculated and experimental B(E2) values (in e^2b^2) and B(E2) ratios

Nucleus	$I_i^+ \to I_f^+$		Exp. B(E2)	Present work	Others B(E2)	Exp. $\frac{B(E2:4^+\to2^+)}{B(E2:2^+\to0^+)}$	Present work
^{158}Dy	2_1	0_1	0.93±0.04	0.937	—	1.37±0.18[e]	1.408
	4_1	2_1	1.27±0.14	1.32	—		
^{160}Dy	2_1	0_1	1.0061[a]	1.029	1.1438[a]	1.40±0.23[e]	1.41
	4_1	2_1	1.4808[a]	1.452	1.6361[a]		
	2_2	0_1	0.0144[a]	0.0177	0.0304[a]		
	2_2	2_1	0.0279[a]	0.0269	0.0780[a]		
^{162}Dy	2_1	0_1	1.0438[a]	1.039	1.077[a]	1.42±0.10[e]	1.41
	4_1	2_1	1.5108[a]	1.4691	1.538[a]		
	6_1	4_1	1.5737[a]	1.5872	1.694[a]		
^{182}W	2_1	0_1	0.82[c]	0.82	0.84[b]	1.389±0.1[e]	1.40
	2_2	0_1	0.057[c]	0.057	0.022[b]		
	2_3	0_1	0.015[c]	0.091	0.0057[b]		
	2_2	2_1	0.086[c]	0.090	0.056[b]		
	4_1	2_1	1.15[c]	1.15	1.19[b]		
	2_1	2_2	0.045	0.090	—		
	2_1	4_1	2.20	2.08	—		
	0_1	2_1	4.15	4.09	—		
^{184}W	2_1	0_1	0.72	0.755	0.670[b]	1.353±0.097[e]	1.40
	2_2	2_1	0.05	0.025	0.0641[b]		
	2_1	4_1	1.98	1.91	—		
	4_1	2_1	1.03	1.06	0.949[b]		
	2_2	0_1	0.025	0.016	0.033[b]		
^{186}W	2_1	0_1	0.51	0.645	0.804[b]	1.304±0.090[e]	1.40
	2_1	4_1	1.62	1.628	—		
	4_1	2_1	0.905	0.905	0.746[b]		
^{158}Gd	2_1	0_1	0.99	0.968	0.100[d]	1.32±0.13[e]	1.41
	2_2	0_1	0.0169	0.0169	0.0236[d]		
	2_3	0_1	0.0016	0.0021	0.0019[d]		
	2_3	4_1	0.0071	0.0078	0.0034[d]		
	4_3	2_1	0.0040	0.0025	0.0047[d]		
	4_3	4_1	0.0022	0.0031	0.0015[d]		
	2_2	2_1	0.029	0.026	0.0039[d]		
	2_2	4_1	0.0013	0.0017	0.0019[d]		
	3_1	2_1	0.0297	0.0295	0.0043[d]		
	3_1	4_1	0.0177	0.015	0.0021[d]		

[a]Reference [8] [b]Reference [12] [c]Reference [13] [d]References [14] [e]Reference [15]

term P^+P was, therefore, added to the SU(3) Hamiltonian for the energy level of $^{158-162}Dy$ nuclei to be in better compatibility with the experimental data. This term in the Hamiltonian , took the different variable values for every isotope than the values which were obtained theoretically in the earlier studies. By adding this term, the disagreement occuring in the β-band of $^{158-162}Dy$ isotopes was totally removed and perfect agreement was obtained with the experimental data. The low-lying energy levels of $^{182-186}W$ on ^{158}Gd nuclei were calculated by the same method and good agreement was also shown.

The ratio of $a_0/4\,a_2$ which was approximately -1 in largely deformed nuclei took the values of - 0.242, - 0.816, - 0.645, - 0.053, - 0.456, - 0.820, and - 0.0422 in ^{158}Dy, ^{160}Dy, ^{162}Dy, ^{182}W, ^{184}W, ^{186}W, and ^{158}Gd respectively. According to this values $^{160-162}Dy$, ^{186}W nuclei are deformed remarkably and they are closer to O(6) limit . However, if a_0P^+P terms were larger in magnitude, the structure of O(6) would have been dominant for the other nuclei. Since this term was small, the symmetry was close to SU(3) rather than O(6). The values of B(E2) were calculated for all of these nuclei for which the levels of energy were already determined. These calculated values were in good agreement with the experimental results of the previous studies [8, 12, 15]. Moreover, these calculated B(E2) values were closer to the experimental results than those obtained employing theoretical approaches.

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